American University of Beirut
Department of Electrical and Computer Engineering
EECE 290 – Analog Signal Processing
Quiz I - Solution

Problem 1(4 pts)

Why do we use Laplace transform in electrical circuits?

a) To transform signal of time domain to frequency domain.
b) To transform signal of time domain to s-domain.
c) To facilitate the complex differential calculations.
d) a and c.
e) all of the above.

If an input signal is applied to the inverting input of an op-amp with the noninverting input grounded, the output signal would have:

a) Same polarity with the input.
b) Opposite in polarity with the input.
c) Neutral polarity since the one of the inputs is grounded.
d) Cannot be determined without a numerical value of both inputs.
e) None of the above.

Reactive power:

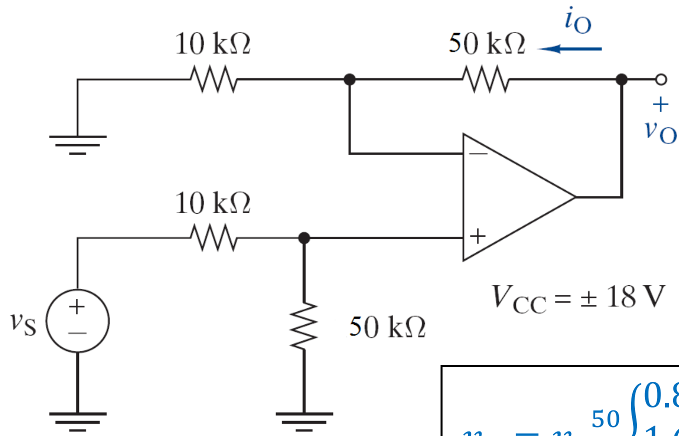
a) Surges back and forth between electric devices.
b) It is stored in magnetic and electric fields.
c) Only occurs when E and I are out of phase.
d) All of the above
e) None of the above

In the power industry a common practice is to use parallel inductive elements to decrease the power factor just like parallel capacitors are used to increase the PF.

a) True
b) False

Problem 2 (4 pts)

Find i_0 for v_S= 1 V. The Op-Amp is operating in its linear mode.



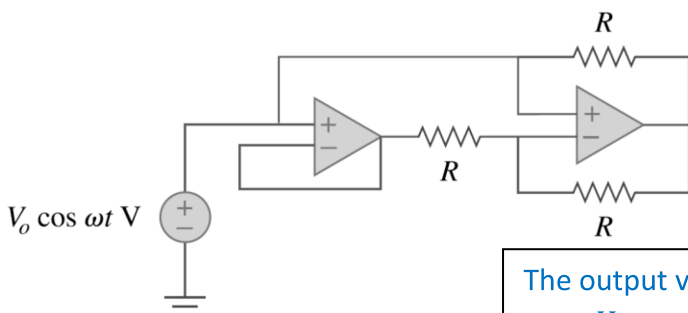
$$v_+ = v_S \frac{50}{60} \begin{cases} 0.833 \text{ V} \\ 1.667 \text{ V} \\ 2.5 \text{ V} \end{cases}$$

$$i_o = \frac{v_+}{10} = \begin{cases} 0.0833 \text{ mA} \\ 0.1667 \text{ mA} \\ 0.250 \text{ mA} \end{cases}$$

a. 0.0333 mA
b. 0.1667 mA
c. 0.253 mA
d. 0.0833 mA
e. None of the above (specify your answer) ______________________________

Problem 3 (3 pts)

Find the total average power absorbed by the resistors. Both Op-Amp are operating in their linear modes.



The output voltage of op-amp 1 is:
$v_1 = V_o \cos \omega t$

The output voltage of op-amp 2 is also:
$v_2 = V_o \cos \omega t$

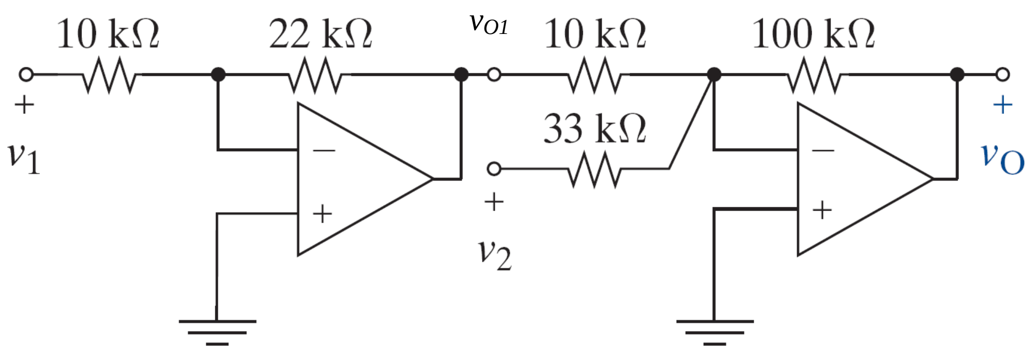
No current flows in the resistors and their total power is 0.

a. $V_0^2/2R$
b. $V_0^2/3R$
c. $V_0^2/4R$
d. $V_0^2/6R$

e. None of the above (specify your answer) ______________________

Problem 4 (4pts)

Consider the following op-amp circuit where v_1= 1 V and v_2= 2 V



1. Determine the output voltage v_{O1} of the first amplifier. (2pts)

$$v_- = v_+ = 0\text{V} \quad \frac{v_{o1}}{22} + \frac{v_1}{10} = 0 \Rightarrow v_{o1} = -\frac{22v_1}{10} = -2.2\text{V}$$

   a. -3.24 V
   b. -1.32 V
   c. -2.2 V
   d. -1.76 V
   e. None of the above (specify your answer) ________________________

2. Determine the output voltage v_O of the op-amp circuit. (2 pts)

   a. 10.87 V
   b. 12.43 V
   c. 16.81 V
   d. 15.94 V
   e. None of the above (specify your answer) ________________________

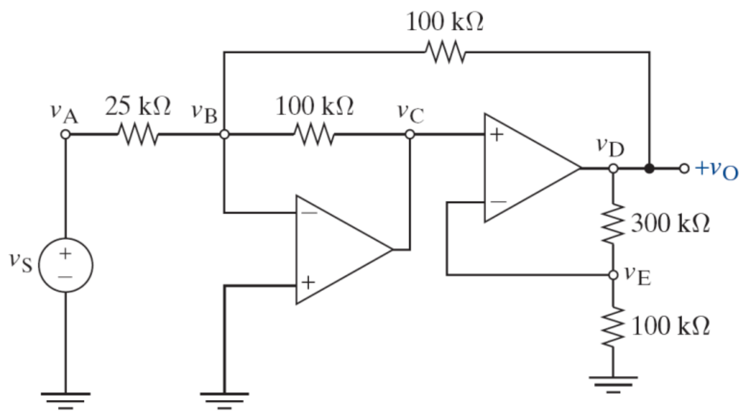
KCL at negative node of op-amp:

$$\frac{v_o - 0}{100} + \frac{v_{o1} - 0}{10} + \frac{v_2 - 0}{33} = 0 \Rightarrow$$

$$v_o = -\left(\frac{v_{o1}}{10} + \frac{v_2}{33}\right) \times 100 = \begin{matrix} 15.94\text{V} & \text{A} \\ 9.88\text{ V} & \text{B} \\ 3.818\text{ V} & \text{C} \end{matrix}$$

Problem 5 (4 pts)

Consider the following op-amp circuit. Both Op-Amp are operating in their linear modes.



1. Determine a relationship between the output voltage v_O and the voltage v_C. (2 pts)

$$v_- = v_+ = 0\text{V} \quad \frac{v_{o1}}{22}+\frac{v_1}{10}=0 \Rightarrow v_C = v_E = \frac{100v_o}{400} = \frac{v_o}{4}$$

2. Determine the output voltage v_O in terms of v_S. (2 pts)

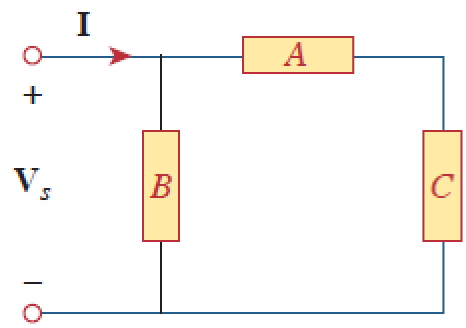
KCL at negative node of op-amp:

$$\frac{v_s-0}{25}+\frac{v_o-0}{100}+\frac{v_o-0}{4\times100}=0 \Rightarrow$$

$$v_o = -\frac{100}{25\times\left(1+\frac{1}{4}\right)}v_s = -3.2\,v_s$$

Problem 6 (6 pts)

In the circuit shown below, load A receives 4 KVA at 0.8 pf lagging, device B receives 3 kVA at 0.4 pf leading, while device C is inductive and consumes 1 KW and receives 500 VAR.



1. Determine the complex power (S_A, S_B, S_C) received by each load, and the total complex power S_T received by the entire system. (3 pts)

$$S_A = 4000 \times 0.8 + j4000 \times 0.6 = 3200 + j2400 \text{ VA}$$
$$S_B = 3000 \times 0.4 + j3000 \times 0.9165 = 1200 - j2750 \text{ VA}$$
$$S_C = 1000 + j500 \text{ VA}$$
$$S_T = S_A + S_B + S_C = 5400 + j150 \text{ VA}$$

A

$$S_A = 4000 \times 0.8 + j4000 \times 0.6 = 3200 + j2400 \text{ VA}$$
$$S_B = 3000 \times 0.4 + j3000 \times 0.9165 = 1200 - j2750 \text{ VA}$$
$$S_C = 1000 + j2500 \text{ VA}$$
$$S_T = S_A + S_B + S_C = 5400 + j2150 \text{ VA}$$

B

$$S_A = 4000 \times 0.8 + j4000 \times 0.6 = 3200 + j2400 \text{ VA}$$
$$S_B = 3000 \times 0.4 + j3000 \times 0.9165 = 1200 - j2750 \text{ VA}$$
$$S_C = 1000 + j5000 \text{ VA}$$
$$S_T = S_A + S_B + S_C = 5400 + j4650 \text{ VA}$$

C

2. Determine the power factor of the entire system. (1 pt)

$\theta = \tan^{-1}(150/5400) = 1.59°$

$PF = \cos 1.59° = 0.9996$ A

$\theta = \tan^{-1}(2150/5400) = 21.71°$

$PF = \cos 21.71° = 0.929$ B

$\theta = \tan^{-1}(4650/5400) = 40.73°$

$PF = \cos 40.73° = 0.7578$ C

3. Find **I**, given that $\mathbf{V_s}$= 240<45^0 V (rms). (2 pt)

$S = VI^*$

$I^* = \frac{5402<1.59°}{240<45°} = 22.5 < -43.41°\text{ A}$

$I = 22.5 < 43.41° = 16.35 + j15.46\text{ A}$ A

$S = VI^*$

$I^* = \frac{5812<21.71°}{240<45°} = 24.2 < -23.29°\text{ A}$

$I = 24.2 < 23.29° = 22.23 + j9.57\text{ A}$ B

$S = VI^*$

$I^* = \frac{7126<40.73°}{240<45°} = 29.7 < -4.27°\text{ A}$

$I = 29.7 < 4.27° = 29.61 + j2.21\text{ A}$ C

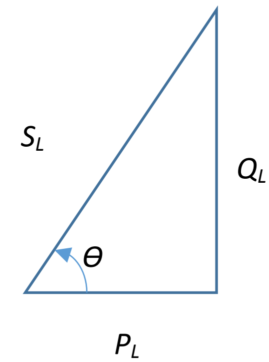
Problem 7 (4 pts)

The lighting and motor loads of a small factory establish a 10 kVA power demand at a 0.6 lagging power factor on a 208 V, 60 Hz supply.

1. Establish the power triangle of the factory load. Draw and label each component of this triangle. Show your calculations. (2 pts)

$$P_L = 10{,}000 \times 0.6 = 6{,}000 \text{ W}$$
$$\theta = \cos^{-1}(0.6) = 53.13°$$
$$Q_L = P_L \tan\theta = 8{,}000 \text{ W}$$
$$S_L = \sqrt{P_L^2 + Q_L^2} = 10{,}000 \text{ VA}$$

A



$$P_L = 10{,}000 \times 0.7 = 7{,}000 \text{ W}$$
$$\theta = \cos^{-1}(0.7) = 45.57°$$
$$Q_L = P_L \tan\theta = 7{,}141 \text{ W}$$
$$S_L = \sqrt{P_L^2 + Q_L^2} = 10{,}000 \text{ VA}$$

B

$$P_L = 10{,}000 \times 0.8 = 8{,}000 \text{ W}$$
$$\theta = \cos^{-1}(0.8) = 36.87°$$
$$Q_L = P_L \tan\theta = 6{,}000 \text{ W}$$
$$S_L = \sqrt{P_L^2 + Q_L^2} = 10{,}000 \text{ VA}$$

C

2. Determine the value of a capacitor that should be placed in parallel with the load to raise the power factor to unity. (2 pts)

Note division by 0.5 is needed for no rms values

T

$$Q_C = V^2/X_C \qquad X_C = \frac{208^2}{8000} = 6.06\ \Omega$$
$$C = 1/(2\pi \times 60 \times 5.4) = 490.5\ \mu\text{F}$$

A

$$Q_C = V^2/X_C \qquad X_C = \frac{208^2}{7141} = 5.4\Omega$$
$$C = 1/(2\pi \times 60 \times 6.06) = 437.1\ \mu\text{F}$$

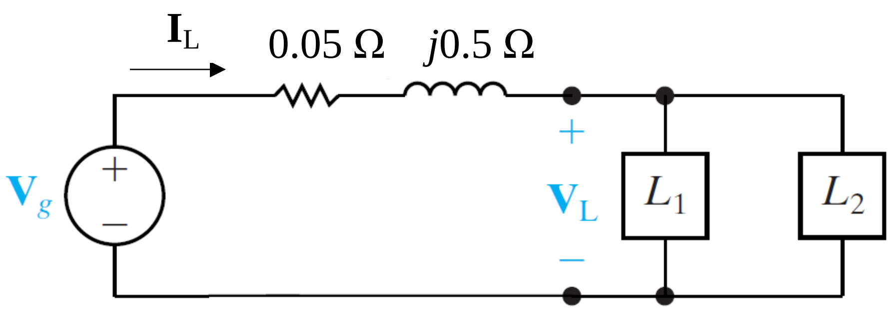
B

$$Q_C = V^2/X_C \qquad X_C = \frac{208^2}{6000} = 7.21\ \Omega$$
$$C = 1/(2\pi \times 60 \times 7.21) = 367.4\ \mu\text{F}$$

C

Problem 8 4(pts)

Load L_1 absorbs an average power of 10 kW at a 0.9 power factor leading; Load L_2 has an impedance of $60 + j80\ \Omega$. The voltage at the terminals of the loads is $1000\sqrt{2}\cos 100\pi t$ V.



1. Find the rms value of the total load current $\mathbf{I}_L$ in amperes. (2 pts)

$S_1 = 10 - j4$ KVA $S_2 = V^2/Z^* = (1000)^2/(60 - j80) = 6 + j8$ kVA $S_L = S_1 + S_2 = 16 + j4$ kVA $I_L = S_L^*/V^* = (16 + j4)^*/1 = 16 - j4$ A	A
$S_1 = 10 - j4$ KVA $S_2 = V^2/Z^* = (1100)^2/(60 - j80) = 7.26 + j9.68$ kVA $S_L = S_1 + S_2 = 17.26 + j5.68$ kVA $I_L = S_L^*/V^* = (17.26 + j5.68)^*/1.1 = 15.69 - j5.16\ A$	B
$S_1 = 10 - j4$ KVA $S_2 = V^2/Z^* = (1200)^2/(60 - j80) = 8.64 + j11.52$ kVA $S_L = S_1 + S_2 = 18.64 + j7.52$ kVA $I_L = S_L^*/V^* = (18.64 + j7.52)^*/1.2 = 15.53 - j6.27$ A	C

2. Calculate the rms voltage V_g of the source. Does the load voltage lead or lag the source voltage and by how many degrees? What is the active and reactive power supplied by the source? (2 pts)

$V_g = V_L + I_L Z_{LINE} = 1000 + (0.05 + j0.5)(16 - j4)$ A

$= 1002.8 + j7.8$ V $= 1002.8 \angle 0.446°$ V

The load voltage lags the source voltage by 0.446°.

The active and reactive power supplied by the source are:

$S_g = V_g I_L^* = (1002.8 + j7.8)(16 + j4) = 16013 + j4136$ VA

$V_g = V_L + I_L Z_{LINE} = 1100 + (0.05 + j0.5)(15.69 - j5.16\ A)$ B

$= 1103.4 + j7.59 = 1103.4 \angle 0.394°$ V

The load voltage lags the source voltage by 0.394°.

The active and reactive power supplied by the source are:

$S_g = V_g I_L^* = (1103.8 \angle 0.394°)(15.69 + j5.16\ A) = 17273 + j5812$ VA

$V_g = V_L + I_L Z_{LINE} = 1200 + (0.05 + j0.5)(15.53 - j6.27\ A)$ C

$= 1103.9 + j7.45 = 1203.9 \angle 0.355°$ V

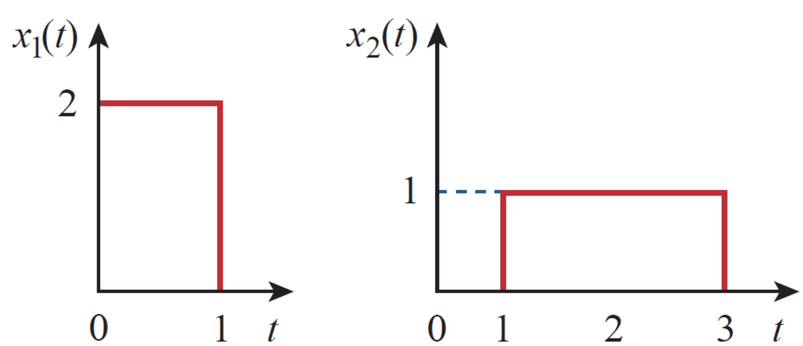
The load voltage lags the source voltage by 0.355°.

The active and reactive power supplied by the source are:

$S_g = V_g I_L^* = (1203.9 \angle 0.355°)(15.69 + j6.27\ A) = 18650 + j7664$ VA

Problem 9 (6 pts)

Consider the two signals $x_1(t)$ and $x_2(t)$ below:



1. Write x_1(t) and x_2(t) in terms of step functions. (2 pts)

$$x_1(t) = 2[u(t) - u(t-1)]$$

$$x_2(t) = [u(t-1) - u(t-3)]$$

2. Determine the Laplace transform $X_1(s)$ of x_1(t) and $X_2(s)$ of $x_2(t)$. (2 pts)

$$X_1(s) = \frac{2}{s} - 2\frac{e^{-s}}{s}$$

$$X_2(s) = \frac{e^{-s}}{s} - \frac{e^{-3s}}{s}$$

3. Let $Y(s) = X_1(s) \cdot X_2(s)$. Determine $y(t)$ by the inverse Laplace transform of $Y(s)$. (2 pts)

$$X_1(s)X_2(s) = \frac{2e^{-s}}{s^2} - 2\frac{e^{-3s}}{s^2} - 2\frac{e^{-2s}}{s^2} + 2\frac{e^{-4s}}{s^2}$$

$$y(t) = 2(t-1) - 2(t-3) - 2(t-2) + 2(t-4)$$

Problem 10 (4 pts)

The Laplace transform function representing the output voltage of a network is expressed as

$$V_0(s) = \frac{120}{s(s+10)(s+20)}$$

1. Determine the value of this voltage at t= 1 s. (Hint solve for $v_0(t)$) (3 pts)

$$V_0(s) = \frac{120}{s(s+10)(s+20)} = \frac{K_1}{s} + \frac{K_2}{s+10} + \frac{K_3}{s+20}$$

For s = 0, K_1= 3/5

For s = -10, K_2= -6/5

For s = -20, K_3= 3/5

Using Laplace inverse:

$$v_0(t) = \left(\frac{3}{5} - \frac{6}{5} * e^{-10t} + \frac{3}{5} * e^{-20t}\right) * u(t)$$

For Version B the gain is 240, so multiply all coefficients by 2

For Version B the gain is 360, so multiply all coefficients by 3

2. Find the final value of the voltage <u>without</u> using $v_0(t)$. (1 pts)

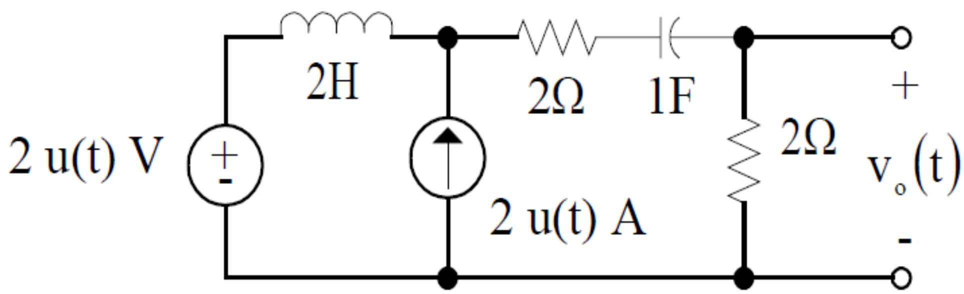
   Using final value theorem: $v_0(\infty) = s\,V_0(s)$ at $s = 0 = \frac{120}{(10*20)} = 0.6V$

   For Version B the final value is 1.2V

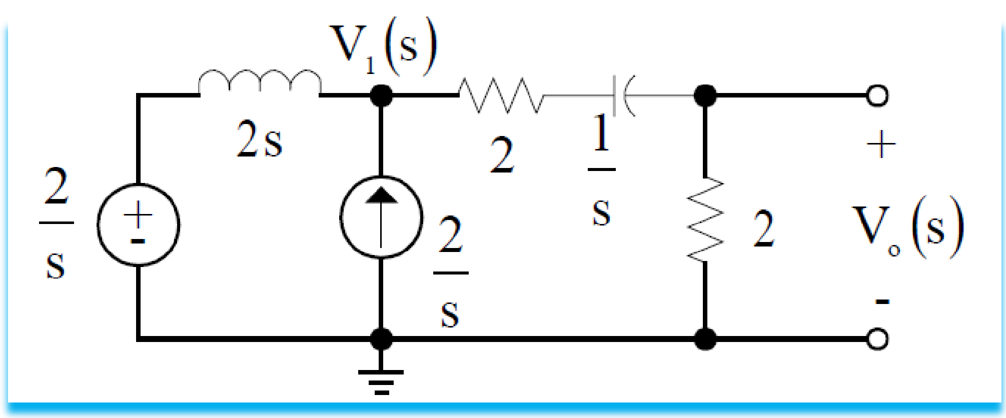
   For Version C the final value is 1.8 V

<u>Problem 11 (7 pts)</u>

Consider the following circuit diagram where it is required to determine the voltage $v_0(t)$ for $t > 0$, with the initial conditions being zero.



1. Draw the s-domain equivalent circuit (2 pt)



2. Solve for the s-domain voltage $V_0(s)$ (2pts)

   Write KCL at node V_1

$$\frac{V_1 - \frac{2}{s}}{2s} - \frac{2}{s} + \frac{V_1}{4 + \frac{1}{s}} = 0$$

Solve for V_1

$$V_1 = \frac{2(2s+1)(4s+1)}{s(2s^2+4s+1)}$$

V_o is obtained from V_1 by voltage division

$$V_o = V_1\left(\frac{2}{4+\frac{1}{s}}\right) = \frac{4(2s+1)}{2s^2+4s+1} = \frac{A}{s+0.9} + \frac{B}{s+1.71}$$

3. Obtain a time-domain expression for the voltage $v_0(t)$ (3 pts)

$$V_o = \frac{4(2s+1)}{2s^2+4s+1} = \frac{A}{s+0.9} + \frac{B}{s+1.71}$$

$$v_o(t) = (0.59e^{-0.29t} + 3.41\mathrm{e}^{-1.71t})\, u(t)\ \text{V}$$